

ITERATIVE METHODS FOR SOLVING NONLINEAR EQUATIONS

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In this work, four new iterative methods were developed to solve a nonlinear equation of one variable. The initial two-step iterative method based on Cubic Spline functions has been derived, and it has been improved to a four-step iterative method using the predictor-corrector method. The Newton Raphson method is combined with second-order Taylor expansion of the nonlinear function is used as the predictor, and the initial iterative scheme using Cubic Spline functions is used as the corrector. The order of convergence of two iterative schemes is analyzed, and it has been proved that the two-step iterative method has a third-order convergence, and the four-step iterative method has a twelfth order convergence. The methods have been tested for various highly nonlinear equations, and the results have been compared with the well-known Newton Raphson method. The third and fourth four-step iterative schemes are derived based on decomposition techniques. Two iterative methods have tenth and nineteenth order of convergence, respectively, which both are higher than the order of convergence of the Newton Raphson method. The iterative methods are implemented by using MATLAB for some highly nonlinear problems with the accuracy of 10^{-15} . The number of iterations to converge to the approximate solutions with the same initial conditions have been compared. It has been proved that all proposed iterative methods are applicable for the nonlinear equation that occurs in the theory of single-slit diffraction, which fails to converge when the Newton Raphson method is used. According to the results, all proposed iterative schemes are better than the Newton Raphson method in terms of efficiency and the order of convergence.

Keywords: Convergence analysis, Decomposition techniques, Iterative methods, Nonlinear equations, Spline functions