

**THREE NEW HIGHER ORDER ITERATIVE SCHEMES FOR SOLVING  
NON-LINEAR EQUATIONS**

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Three new higher order iterative schemes have been developed for solving non-linear equations of one variable. Initial two-step iterative scheme has been derived by combining the Newton-Raphson method and Simpson's Rule. Based on this initial scheme, the first two-step scheme has been developed using the second order Taylor's expansion and the improved Newton-Raphson method. Combining the Predictor-Corrector method with above schemes, the second scheme with three steps has been derived. The third three-step scheme was derived by combining the second scheme with the third order polynomial interpolation approximation of the second derivative. Convergence analysis of proposed schemes have been carried out, and it has been proved that first, second and third iterative schemes have order of convergence four, five and five, respectively. Hence, we have established that the new schemes have higher orders of convergence than that of the Newton-Raphson method. The performance of proposed schemes has been simulated, for a set of highly non-linear equations for a different set of initial guesses. The number of iterations taken by each scheme to converge the approximate solution with  $10^{-10}$  accuracy has been calculated, and new schemes converge rapidly as compared to the Newton-Raphson method. The number of iterations taken by each scheme decreases with the rise of order of convergence. In some cases, when the initial guesses are far away from the actual solution, the Newton-Raphson method runs into an infinite-loop and fails to converge. However, proposed schemes converge to the solution of these problems without falling into infinite-loops. Absolute errors in approximate solutions in the iterative processes are compared graphically, and they are in accordance with the derived orders of convergence of schemes and the Newton-Raphson method. Further, the computational orders of convergence of the schemes are compared with the Newton-Raphson method. The values are in accordance with the orders of convergence analysed.

**Keywords:** Computational order, Convergence analysis, Higher order iterative method, Non-linear equations, Predictor-Corrector method